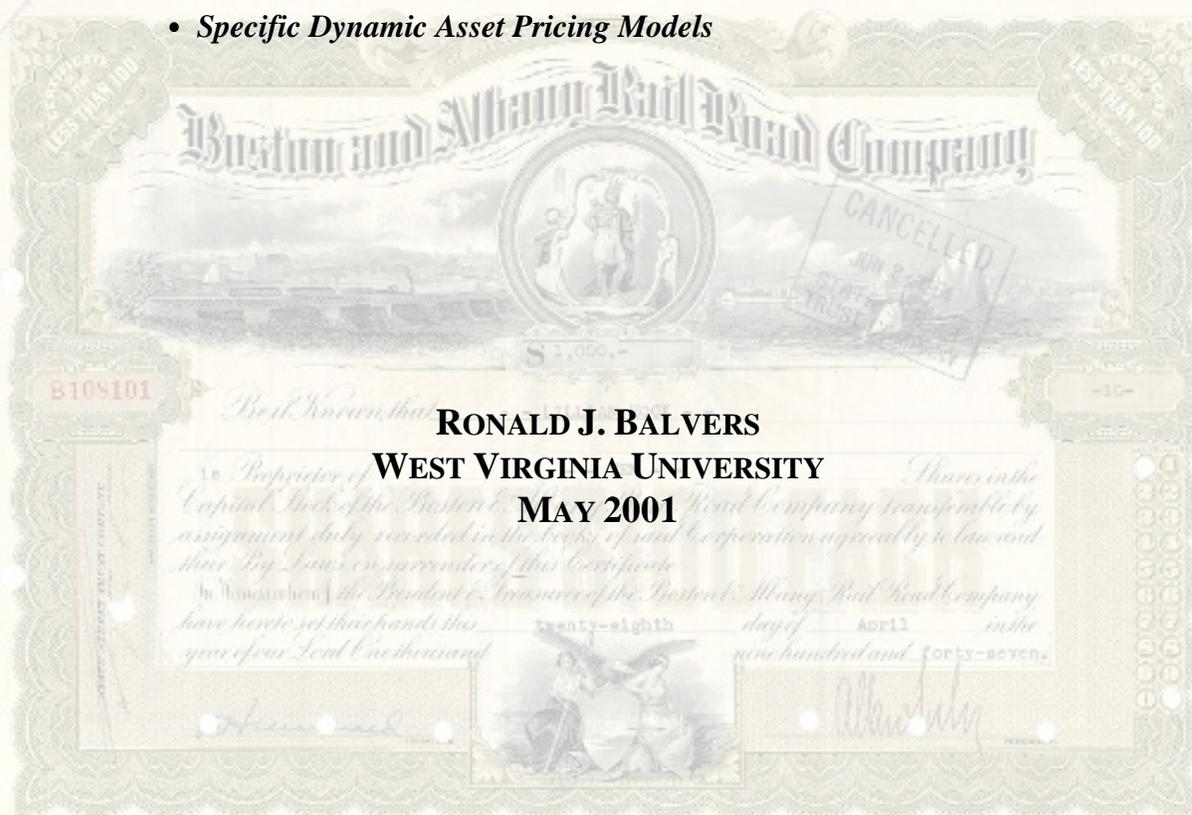


Foundations of Asset Pricing

- Preliminaries
- Mean-Variance Portfolio Choice
- Basic of the Capital Asset Pricing Model
- Static Asset Pricing Models
- Information and Asset Pricing
- Valuation in Complete Market Economies
- General Issues in Valuation and Arbitrage
- Dynamic Asset Pricing–General Models
- Specific Dynamic Asset Pricing Models



Preface

This manuscript is based on my lecture notes for the second course in the three-course financial economics sequence taught in the Ph.D. program in economics at West Virginia University. This course only covers theoretical models of and empirical approaches to asset pricing. The first course in the full financial economics sequence deals with topics in the economics of risk and uncertainty and portfolio theory. Accordingly, the material presented here assumes knowledge of basic concepts in risk theory such as measures of relative and absolute risk aversion and the Rothschild-Stiglitz concept of increases in risk. Portfolio theory is only summarized briefly in Chapter II as an introduction to the Capital Asset Pricing Model. Additionally, no options and futures-pricing material and no corporate finance issues are covered since these are taught in the third course of the financial economics sequence.

The material is developed for students with a strong background in economics but not necessarily with a lot of exposure to finance at the undergraduate level. Some of the material, in particular the material in the first three chapters, is standard in investments courses and in beginning graduate-level asset pricing courses. Much of the material is not standard in these courses, and quite a bit of it has been developed by me—so be careful and check all results!

Much of the theory in asset pricing is developed in continuous time. For economists the continuous-time approach is often counter-intuitive due to the nature of the approach as well as the fact that most economic theory these days is developed in discrete time. For this reason, all models in this manuscript are presented in discrete time. To be able to do so, in many cases a generalization of Stein's Lemma (presented and proven in Appendix C) is employed; to my knowledge, this generalization has not been used elsewhere. Other topics that are covered here in a novel way are: the general equilibrium perspective on the CAPM in Chapter III; the CAPM with multiple consumption goods and the international CAPM in Chapter IV; the cross-sectional asset pricing implications of asymmetric information in Chapter V; the investment-based asset pricing model and the conditional CAPM in Chapter IX.

There are a host of very good textbooks in asset pricing. In particular, Campbell, Lo, and MacKinlay (1997), Cochrane (2001), and the less recent Huang and Litzenberger (1988). Chapter VI in these notes is partly based on two chapters in Huang and Litzenberger and Chapter VII is my summary of some of the issues introduced into the mainstream of asset pricing theory by Cochrane. There are many issues covered in these textbooks that are not covered here, especially in the area of econometric techniques applied in financial economics (the perspective I take is that simpler econometric techniques are more robust to the deviations from ideal statistical conditions likely to exist in real data). These notes are best seen as a good way of bridging the gap between basic material in investments and the more advance material in Campbell, Lo, and MacKinlay (1997), Cochrane (2001), and Huang and Litzenberger (1988).

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